Comparison of fractal and empirical model for estimation Soil Water Retention Curve

Mahdi Shorafa[A], Mahmood Fazeli[A*], Davood Namdar Khojasteh[A]

[A]Departmen of Soil Science, University of Tehran, Karaj, Iran, [*] Email: mahmoodfazelisangani@gmail.com

ABSTRACT
Many empirical and fractal models have been developed to describe the soil water retention curve (SWRC). In this study, the fractal dimension of soil texture, was used instant of the fractal dimension of SWRC in Tyler and Wheatcraft (1990) model; the estimated results being compared with experimental data for verification and compared with the estimated results from Campbell (1974) empirical model. Results showed a reasonably good estimation of soil water retention curves for the most soils by both fractal and empirical models. Also the results showed that there is not significant difference between empirical and fractal models in estimating SWRC, when the fractal dimension of SWRC estimated by soil texture fractal dimension.

Key words
Empirical model, fractal model; soil texture; soil water retention curve**.**

Introduction
The soil water retention curve (SWRC) is one of the important hydraulic functions for modeling flow transport in porous media. Due to difficulties and labor costs when measuring SWRC, it has become necessary to develop methods to describe the function utilizing readily available data, such as soil texture. Many empirical models for SWRC have been developed (Brooks and Corey 1964; van Genuchten 1980; Russo 1988). In these models parameters were usually estimated by fitting the functions with measured data, and the pedotransfer functions (PTFs) were used empirically to describe the relationship between the parameters and basic soil data (Minasny *et al.* 1999; Wosten *et al.* 2001). Based on the assumption that either the soil solid phase or the soil void space has affine self-similarity, the soil phases can be described using the fractal scaling theory. Several models have been derived either using the fractal nature of the solid or void phases or both (Tyler and Wheatcraft 1990; Rieu and Sposito 1991a; Perrier *et al.* 1996; Perfect *et al.* 1999). Tyler and Wheatcraft (1990) applied the Sierpinski Carpet (Mandelbrot 1983) to describe the soil pore size distributions and developed a power-law form for SWRC, similar to the functions of Brooks and Corey (1964) and Campbell (1974). Perfect *et al.* (1999) developed SWRC models, which were in a power-law form but differed from Campbell (1974) models. Fractal dimensions of the solid matrix (i e., soil particle size distribution and soil texture) and the void phase (i.e., soil pore size distribution and soil pore surface) can characterize the fractal nature of soils. The objective of this study was to determine the fractal dimension of soil texture and replace it with the fractal dimension of SWRC for its estimation and comparing the results with the Campbell (1974) empirical model.

Methods
The used fractal and empirical Models
the Campbell (1974) model was used as an empirical model that express by equation (1):

$$\frac{\psi}{\psi_a} = \left(\frac{\theta}{\theta_s}\right)^{-b_c} \tag{1}$$

The fractal model was used in this study was the Tyler and Wheatcraft (1990) model that express by Eq. (2):

$$\theta = \theta_s\left(\frac{\psi}{\psi_a}\right)^{D_M-3} \tag{2}$$

Where ψ, is the capillary tension head (cm) and θ is the soil water content (cm3 crn3), θ_s, is the saturated soil water content (cm3 crn3), ψ_a is the air entry pressur (cm), $\boldsymbol{D_m}$ is the fractal dimension of SWRC, and b_c, is an empirical coefficient.

Samples and measurements
Experimental data of texture, and soil water retention for 40 soils collected at different places in Iran were used to estimate the fractal dimension of SWRC by the mass fractal dimension of soil texture and water

retention curve. The soils, cover all range of texture classes. Undisturbed samples were taken directly from each soil sample site was used for measuring SWRC. Also, 500 g of disturbed soil samples were collected from each sample site for determining soil texture according to the US Department of Agriculture (USDA) texture classification standards. Soil water retention data were measured using the pressure plate apparatus at seven metric potentials (100, 300, 1000, 3000, 5000, 10000, 15000 cm).

Determination soil texture fractal dimension

The texture (Sand, silt and clay) data and the log-transformed form of the Tyler and Wheatcraft (1992) equation were used to determining the fractal dimension D; By employing a linear regression between the cumulative log mass fractions and log characteristic particle radius(R) for all soils, the fractal dimension D, was then determined. Tyler and Wheatcraft (1992) equation express as:

$$\frac{M(r<R)}{M_{\mathrm{T}}}=\left(\frac{R}{R_{\max}}\right)^{3-D_{\mathrm{m}}} \quad (3)$$

where r is the grain size, R represents the characteristic particle radius, M (r < R) is referred to as the mass of grain radius r less than R, MT is the total mass, R, is the maximum characteristic particle radius, and D, is the mass fractal dimension.

Models verification and comparison

The estimated soil water retention curves were compared with the measured data, and the difference between the estimated soil water retention curves and the measured data was then quantified by using the Mean absolute error (MAE) and Mean Square error (MSE). Linear regression was then performed between measured and estimated water content for all soils and coefficients of determination (r^2), was determined. The MAE, MSE and r^2 were used for models comparison.

Results

Figure 1 shows the results of a linear regression between the cumulative log mass fractions and log characteristic particle radius for a typical soil: soil 6 (clay). The mass fractal dimension determined with soil texture data, ranged between 2.95 to 2.61 for clay and sandy soil textures. Tyler and Wheatcraft (1992) using the soil mass distribution, Rieu and Sposito (1991b) using the aggregate size in the three-dimensional Euclidian domain have found that the fractal dimension of soils were in the range of 2 to 3.
Comparing all data of measured soil water content versus the estimated by using models showed a reasonably good estimation of soil water retention curves for the most soils. Fractal model with using mass fractal in it, showed a better estimation for light textured soils than the other soils. The estimated results compared with the measured data having mean absolute errors less than 0.05 for over 63% and 67%of the measurements, for fractal and empirical model respectively.

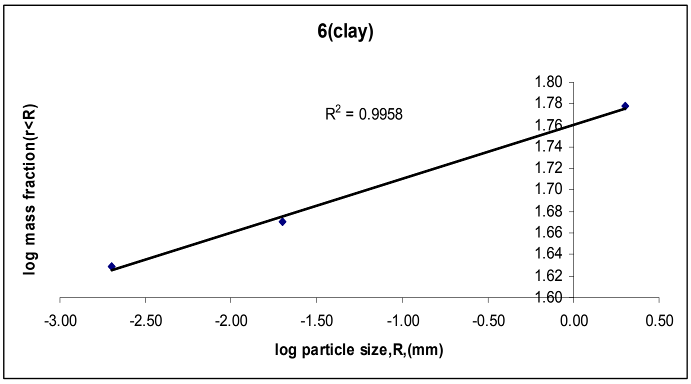

Figure 1. Linear regression between the cumulative log mass fractions and log characteristic particle radius for soil 6.

Table 1. MAE and the MSE and R 2 obtained form comparing all data of the measured soil water content versus the estimated by using fractal and empirical models.

Model	MAE			MSE			R^2		
	Max	Min	Mod	Max	Min	Mod	Max	Min	Mod
Fractal	0.14	0.01	0.05	0.02	0.0002	0.005	0.99	0.91	0.98
Empirical	0.11	0.01	0.5	0.01	0.0002	0.003	0.99	0.84	0.98

Table 1 shows the MAE and the MSE and R^2 obtained form comparing all data of the measured soil water content versus the estimated by using fractal and empirical models. This results show that there is not significant difference between empiricl, Campbell(1974), and fractal, Tyler and Wheatcraft (1990), models in estimating SWRC, when the fractal dimension of SWRC estimate by soil texture fractal dimension. Estimated and measured SWRC had shown in Figure 2 for three typical soil: soil 7 (sandy clay), soil 20 (silty clay loam) and soil 39 (sandy).

Conclusion

The mass fractal dimension determining with soil texture data, ranged between 2.95 to 2.61 for clay and sandy soil textures. The results indicated that the model with the fractal dimension calculating from the soil texture data was capable of predicting SWRC with reasonable accuracy, especially for light texture soils. Nevertheless it is predicted that it gave better estimated results of soil water content if the fractal dimension of SWRC is estimated in low tension (<1 atm) and high tension(>1) potential separately.

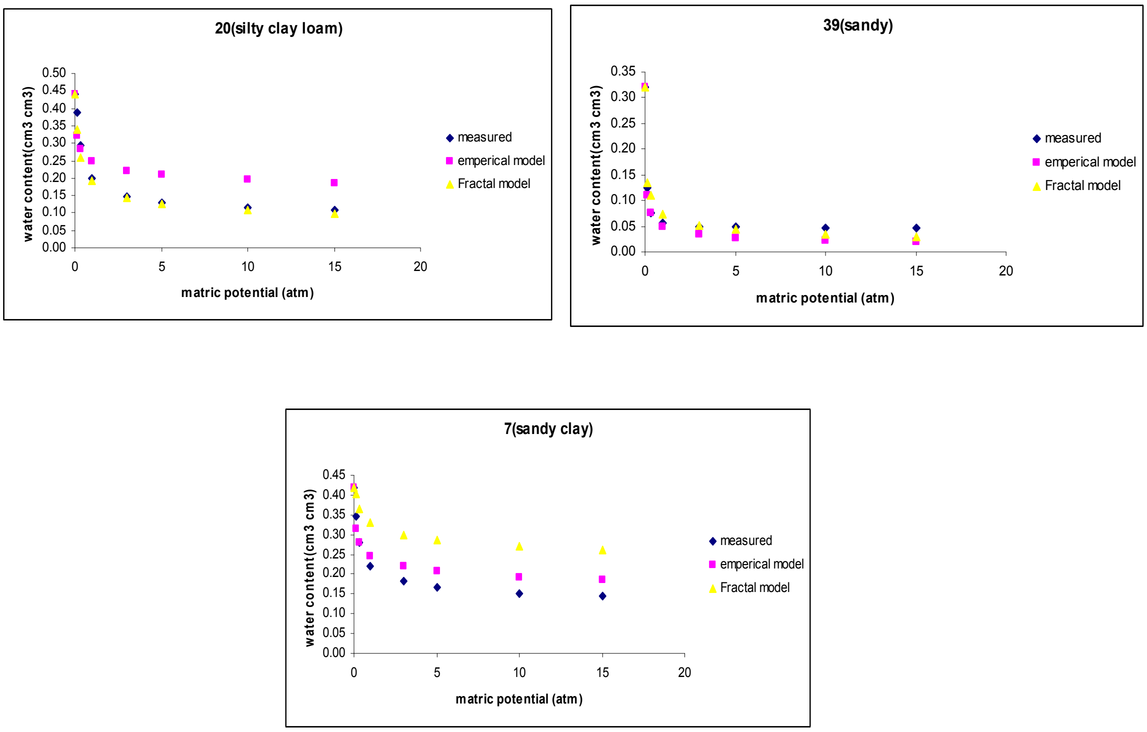

Figure 2. Estimated and measured SWRC had shown in Figure 2 for three typical soil: soil 7 (sandy clay), soil 20 (silty clay loam) and soil 39 (sandy).

The results also showed that there is not significant difference between empirical, Campbell(1974), and fractal, Tyler and Wheatcraft (1990), models in estimating SWRC, when the fractal dimension of SWRC estimate by soil texture fractal dimension.

Reference

Brooks RH, Corey AT (1964) Hydraulic properties of porous media, Hydrol. Pap. 3, Colorado State Univ., Fort Collins, CO.

Campbell GS (1974) A simple method for dekrrninirig unsaturated hydraulic conductivity from moisture retention data. *SoiZ Sci.* **177**, 311-314.

Mandelbrot BB (1983) The Fractal Geometry of Nature. W. H. Freeman, San Francisco. 468pp.

Minasny B, McBratney AB, Bristow KL (1999) Comparison of different approaches to the development of pedotransfer functions for water-retention curves. *Geoderma* **93**, 225-253.

Perfect E (1999) Estimating soil mass fractal dimensions from water retention curves. *Geoderma* **88**, 221-231.

Perrier E, Rieu M, Sposito G, de Marsily G (1996) Models of water retention curve for soils with fractal pore size distribution. *Water Resour. Res.* **32**, 3025-3031.

Rieu M, Sposito G (1991a) Fractal fragmentation, soil porosity, and soil water properties: I. Theory. *Soil Sci. Soc. Am. J.* **55**, 1231-1238.

Rieu M, Sposito G (1991b) Fractal fragmentation, soil porosity and soil water properties: 11. Applications. *Soil* Sci.
Russo D (1988) Determining soil hydraulic properties by parameter estimation: On the selection of a model for the hydraulic properties. *Water Resour. Res.* **24**, 453-459.
Tyler SW, Wheatcraft SW (1990) Fractal processes in soil water retention. *Water Resour. Res.* **26,** 1047-1054.
Tyler SW, Wheatcraft SW (1992) Fractal scaling of soil particle size distributions: Analysis and limitations. Soil
van Genuchten MTh (1980) A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. Soil
Wosten JHM, van Genuchten MTh (1988) Using texture and other soil properties to predict the unsaturated soil hydraulic functions. *Soil Sci. Soc Am. J*. **52**, 1762-1 770.